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ABSTRACT

Preservice teacher preparation programs seek to provide experiences that support this process primarily through clinical and field components. More professional courses include field-based experiences. In addition, interest in and use of cases has been introduced as part of preservice programs to help prospective teachers focus on the dilemmas of teaching throughout their preparation. In this study, variations of case methodology making use of a variety of videotapes were integrated into a methods course for teaching elementary school mathematics. Based on the analysis of the data from five teacher candidates being interviewed twice and a review of selected documents of eight teacher candidates' written work, it indicates that by the end of the course, teacher candidates expressed a desire to be accepting of a variety of student responses and to be open to more than one "right" way of solving problems. Teacher candidates also stressed the importance of using different strategies and probes to "get at" student thinking both in mathematics and other subject areas they will be teaching. Teacher candidates cited the videotaped cases as worthwhile. (Contains 23 references.) (ASK)

A preliminary report

Making Sense of Teaching and Learning Mathematics: Using Videos to Provide "Case-Like" Experiences in an Elementary Mathematics Methods Course

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Introduction

It is becoming increasingly apparent that learning to teach is a developmental process focused on understanding the dilemmas of teaching (Harrington, 1995). Preservice teacher preparation programs seek to provide experiences that support this process primarily through the clinical and field components. However, a number of scholars have questioned the practice of relying totally on these experiences to foster prospective teachers' knowledge (Harrington, 1995). Many of the professional courses that precede the clinical and field components have focused, in the past, on the presentation of techniques and methods. This situation is changing. More professional courses include field-based experiences. In addition, interest in and use of cases has been introduced as part of preservice programs to help prospective teachers focus on the dilemmas of teaching throughout their preparation (Harrington, 1995).

> Merseth (1996) defines a case viewed from a teacher education perspective as:
> ... a descriptive research document based on a real-life situation or event. It attempts to convey a balanced, multidimensional representation of the context, participants, and reality of the situation. It is created explicitly for discussion and seeks to include sufficient detail and information to elicit active analysis and interpretation by users. (Merseth, 1996, p. 726)

Cases are developed to represent reality, to stimulate thought and debate, and, as teaching instruments, for study, examination, and discussion.

While some research on the use of case-based pedagogy in general teacher education courses has been completed (e.g., Levine, 1995; Harrington, 1995), limited research has been conducted on what happens when cases are used as part of the professional development of prospective teachers with respect to teaching mathematics. In addition, little empirical evidence has been developed concerning the effects of video-based case pedagogy in teacher education (Copeland & Decker, 1966) and, by default, in mathematics teacher education. Most recently, narrative case materials have become available that address dilemmas in teaching elementary and middle grades mathematics (Barnett, et. al. 1994., Schifter, 1996 a,b). In addition, other "case-like" materials are now available for supporting reasoning about mathematics instruction and about students' thinking about mathematics; these include teaching vignettes (NCTM, 1991, 1995) and videos (Corwin, 1996, Kamii, 1989, 1990a, 1990b, Richardson, 1990, TERC, forthcoming, CGI, WGBH, 1995) that provide brief windows into models and dilemmas of classroom practice and student understanding in mathematics.

There are a variety of interpretations of what cases are and what the purposes of their use might be. Merseth (1996) provides a simple conceptual framework as an organizing scheme for thinking about cases; the framework divides case purpose and use into three categories.

- Cases as exemplars: This view sees cases as paradigmatic representations of generic problems in teaching (Copeland & Decker, 1996). The emphasis of these cases is on the theoretical, the prescriptive, the model. Their purpose is to develop a knowledge of theory or to build new theories.
- Cases as opportunities to practice analysis and contemplate action: This view sees cases as providing opportunities to practice decision making and problem solving. Cases are used to present situations from which theory emerges rather than explicitly to exemplify.
  > With such cases, students can, within the confines and safety of a teacher education classroom, 'practice such professional skills as interpreting situations, framing problems, generating various solutions to the problems posed and choosing among them.'

Paper presented at AMTE, February, 1997, Washington, D.C.

Cases that focus on problem solving and decision making typically are based on a real situation where 'an actual instance of practice is presented in much of its complexity rather than an episode constructed to illustrate a point.' (Merseth, 1996, p 728)

- Cases as stimulants to personal reflection: In this view, emphasis is on personal professional knowledge and the individual. Reflection is promoted from directly or vicariously experiencing a situation that puzzles or surprises. This view includes the use of teacher-constructed cases along with other types of cases. The cases serve as "the data" and the discussion of the cases articulates possible "courses of action."

Purpose of the Study

The major question under consideration for this study was: what is the impact[1] of using videos to provide "case-like" experiences in a mathematics methods course as part of the professional development program for prospective elementary teachers? While the use of video tapes was not the only instructional strategy employed, it was seen as central to accomplishing the goals of the course (Figure 1).

Figure 1 – Portion of Course Syllabus

EDUC 12 (II) Teaching Mathematics in the Elementary Grades
Cohort II - 11:30 a.m. – 1:00 p.m., Room 310

Logistics
Office Location: 201B Peabody Hall
Email: sfriel@email.unc.edu
Phone: 962-6605

Office Hours: Tuesdays, 1-4 p.m. or by appointment

Overview The traditionally held view of mathematics is that there is a preexisting body of knowledge and that one learns by absorbing information that is transmitted by a more knowledgeable other or text. In contrast, the constructivist (or cognitive) position suggests that all knowledge is constructed and learners create knowledge for themselves by acting on the world through their experiences and activity. The implications of such a view are that, as teachers, we want to provide students with challenging problems which may be explored collaboratively and through class discussion of students' solutions.[2]

The goal of this course is to explore what it means for elementary students to learn (and teachers to teach) mathematics from a constructivist perspective. We will use the NCTM *Curriculum and Evaluation Standards for School Mathematics* and the *NC Standard Course of Study* to help define what students (K-6) need to know and be able to do with respect to mathematics content. We will use the NCTM *Professional Standards for Teaching Mathematics* as a way to focus our thinking and describe teaching that supports and incorporates a constructivist view of learning.

Goals

Mathematics Content Knowledge
Demonstrate knowledge and understanding of the nature of selected mathematics content from the elementary school curriculum and its interaction with a variety of mathematical tools and strategies, including manipulative and visual materials. Topics to be considered:

- Number: prenumeration concepts; numbers (whole numbers, fractions, decimals) and their

[1] For purposes here, impact is defined "loosely" as providing a descriptive summary of teacher candidates' views with respect to content knowledge, pedagogical content knowledge, and knowledge of students during and at the end of the course, and during and at the end of their student teaching experiences.

[2] Wood, T., Cobb, P., Yackel, E., and Dillon, D. (Eds.) (1993.) *Rethinking Elementary School Mathematics: Insights and Issues*. Reston, VA: National Council of Teachers of Mathematics.

relationships; four basic operations with whole numbers and fractions
- Data: concepts of elementary data analysis
- Space: geometric concepts and spatial visualization; measurement concepts and procedures

Knowledge of mathematics curriculum/instructional strategies
- Demonstrate the ability to plan, select, and sequence developmentally appropriate content material within the elementary mathematics curriculum.
- Demonstrate familiarity with and skill in the use of the *North Carolina Standard Course of Study* .
- Demonstrate knowledge and understanding of sources of curriculum (e.g., *North Carolina Standard Course of Study*, professional literature, curriculum guides, school expectations, interests and needs of students, teaching philosophy) and the ability to use those resources to plan and implement developmentally appropriate mathematics instruction.
- Plan and implement mathematics curriculum that addresses assessed needs and abilities of elementary school children.

Knowledge of Students
- Select appropriate mathematical tasks that stimulate students' development of mathematical concepts and skills.
- Demonstrate knowledge and understanding of ways to use assessment to monitor students' mathematical learning and to make instructional decisions.
- Demonstrate knowledge and understanding of ways to use a variety of methods to assess mathematical learning such as open-ended questions, portfolios, and performance tasks.

Book to purchase:

Van De Walle, J. A. (1994). *Elementary school mathematics (second edition).* White Plains, NY: Longman Publishing.

Assignments

1.(15%) Journals/classwork

a. Communication/journaling by Email/computer (once a week):

Each week, a set of questions are provided to which you are asked to respond. My intention is to comment on your thoughts in ways that seem helpful for both of us. This includes your mathematics autobiography.

b. Readings and class-related assignments: This involves a consideration of attendance, preparation, and performance. I will expect that you keep a notebook with this work; I will expect to look at your notebook. I will ask you to evaluate yourself in a similar fashion at the end of the course. I suggest that you use a loose-leaf notebook with the following sections: Syllabus and other handouts from class, Notes from class together with any related handouts, Notes from readings, Video assignments, Journal responses, and other.

2. (25%) Viewing/responding to video situations related to teaching and learning mathematics. For each video, a set of discussion questions also is to be considered. You are expected to prepare a written response to these questions. These assignments may be done as teams of 4; you will turn in "team" lesson plans and discussion question responses. These video tapes not only provide an opportunity to see mathematics lessons about a variety of topics, they show instruction in a variety of classroom settings, including bilingual and urban settings.

3. (20%) Interviewing student/conducting action research mini-project

We will develop plans for doing this work with students as part of our class. The goal is for you to explore students' understanding of some mathematics concept and to focus on "what next" with respect to instruction. Any interview/research you do is limited to a one-time meeting with a student or group of students within a classroom setting and is tied to the ongoing work of the classroom in which you are participating.

Note: the Buschman article is a good example of one kind of "action research".

5. (25%) Unit Plan for teaching a mathematical topic/concept at the grade level (K-6) of your choice. Guidelines for this assignment will be provided. One component will be a database resource file of relevant materials, books, computer software, etc.

6. (15%) Final Examination

The typical protocol for instructional use of cases is to provide appropriate theoretical readings which are followed with the reading of a case designed to provoke discussion of teaching dilemma(s) that require applications of the theory. In using cases, readers first respond individually (usually in writing) to a series of questions used to guide analysis of the case (see Figure 2). A case discussion follows, and readers write additional reflections based on these discussions.

Figure 2: Questions to Focus Reading of Written Case

1. What's going on in this case? Summarize aspects that you think are important to understanding it.
2. What are some of the issues that come up for you in this case? What are some of the questions that this case raises for you?
3. Why do you think it is important to raise the questions, concerns, or issues you wrote about in question (2)?
4. What are some other ways to teach this lesson (set of lessons)?
5. How would you teach this lesson? What materials would you use? What order would you do things in? What examples would you give?
6. What else do you need or want to know about this case?

In this study, variations of case methodology making use of a variety of video tapes (see Figure 3) were integrated into EDUC 12 Methods for Teaching Elementary School Mathematics. The theoretical contexts were developed through the use of readings (primarily from Van De Walle, 1994), a consideration of the developmental sequence of the mathematics content over the K-6 curriculum, and on-going analysis of selected hands-on activities that were explored as ways to teach specific mathematics content. It was the instructor's intent to use the video tapes in each of the three ways detailed by Merseth; questions provided to accompany the viewings were designed to provoke a variety of considerations.

Figure 3: Sequenced Use of Video Materials

Case/Situation	What's this a case of?	Focus of Analysis
Set 1: Looking at lesson and class-room structure • *Marshmallows* (WGBH, Grade 2, Tucson, AZ., bilingual classroom) • *Pumpkin Seeds* (WGBH, Grades 1-2, Boston, MA, inner city classroom) • *Dominoes* (WGBH, Grades 1-2, Boston, MA, inner city classroom) • *Meter Cords* (WGBH, Grades 3-4, Bakersfield, CA)	Purpose: Used to provide opportunities to practice analysis and contemplate action taken and as exemplars of an investigative approach to teaching mathematics. With the exception of *Marshmallows*, each video was viewed outside class by teams of four students. Students carried out the same process with respect to viewing each video. The first two videos were presented as complete examples of classroom investigations. The third video, *Dominoes*, did not have a complete investigative lesson structure. The final tape, *Meter Cords*, was intended to raise issues of quality of lesson structure and student understanding in addition to focus on classroom investigation. Each video tape was linked to a topic being addressed during class.	The first video was viewed by the class together. Teams of 4 teacher candidates were formed; each team was expected to work together on any remaining "outside" class viewings of tapes. In preparation for the first tape, each team selected one of its members to watch and take notes about one of the following: classroom organization; management strategies; what the teacher said and did; and what the students said and did. The context for the tape was established by reading the description provided with the study guide. Once this first tape was viewed, the teacher candidates regrouped themselves, with all teacher candidates (each from a different team) who focused on management meeting together; similarly, three other groups were re-formed based on viewing focus. Each group made a list of points related to the area of focus; these lists were posted and discussed. Following this, a lesson planning structure was introduced and discussed. Students were asked individually to write a complete lesson plan for the video viewed. The remaining three videos were viewed outside class using the same strategies; a team lesson plan was handed in. These tapes, while not discussed explicitly during class, were often referred to as appropriate.

Set 2: Looking at children's thinking and use of interviews as an assessment strategy: Tape: *A look at children's thinking* (Richardson, 1990) Tape: Various *CGI* segments Student conducted interviews	Purpose: Used as stimulant to personal reflection about what children can do and understand with respect to early number concepts and as exemplars of ways to conduct interviews with children Segments of videos were viewed as appropriate for class discussion. For example, after considering counting strategies, the section on counting interviews (Richardson) was viewed. As another example, once CGI problem types were introduced, segments from the videos were used to show students at different stages.	Reading about developmental changes often lacks the impact of actually interviewing students. The video clips gave teacher candidates opportunities to apply what they read. More than once, one or more teacher candidates commented, "Oh, yeah. That's what the book said." Teacher candidates were encouraged to try out interviews with students in their field placements. Eventually, as a course assignment, each teacher candidate had to design and implement an interview with one or more students.
Set 3: Looking at children's thinking in the context of content development and classroom structure • *Calendar Math* (Corwin) • *Developing mathematical ideas* (TERC) (addition - grade 5 segment) • *Multiplication of two-digit numbers* (Kamii) • *Developing mathematical ideas* (TERC) (division)	Purpose: Used as a stimulant to personal reflection about the nature of mathematics and what children can do and as exemplars of ways to support children's development of computation. Both the Corwin video and the TERC segment about addition/subtraction were shown during class. The Kamii tape was viewed by teams of 4 teacher candidates; see Figure 5 for questions. It was discussed in class only as a part of other topics being addressed. The TERC tape (division) was presented over two classes. Student strategies were discussed in detail as part of class review of the tape.	These tapes were viewed in order to highlight the varieties of ways students reason about computation; the ways students use previously learned mathematics concepts to develop new concepts (e.g., addition used to develop multiplication; subtraction and/or multiplication used to develop division); and the importance of developing a mathematics community in which students listen to one another in order to understand mathematics.

Set 4: Looking at initiating lessons linked to the use of manipulative tools • *Cookies to Share* (WGBH, Grade 4, Bakersfield, CA) • *Fractions with Geoboards* (WGBH, Grades 4-5, Tucson, AZ)	Purpose: Used as exemplars of initiating lessons to introduce the concept of fractions. Each tape provided a different visual context for fractions. After *Cookies to Share*, teacher candidates continued to explore fractions using pattern block cookies. After *Fractions with Geoboards*, teacher candidates reviewed a set of lessons that used the geoboard as a model for developing fraction concepts.	Each video provided a model of an introductory lesson that might be considered "elegant" both in its simplicity of structure and in its surfacing of deeper understandings. Each was a case of "less is more." Teacher candidates noted the time allowed to develop a single concept and the presence of a classroom community that fostered discourse.
Set 5: Analyzing the quality of a classroom lesson, taking into account appropriateness of content, pedagogy, and student knowledge • *Arrays and Fractions* (WGBH, Grades 1-3, Shelbyville, KY)	Purpose: Used as an opportunity to practice analysis and contemplate action and as a stimulant to personal reflection. See Figure 6 for questions. This tape raises several instructional issues in terms of pacing, student readiness, teacher direction and appropriate use of models.	Teacher candidates viewed the tape and then were permitted to discuss their reactions as a whole group; the discussion was run by the teacher candidates themselves. They wrote their own individual responses to the questions.

While the teacher candidates were in weekly field placements (i.e., the classrooms in which they would student teach during the spring), it was not possible to rely on their encountering in their placements examples of topics and issues that were being addressed in the course with respect to mathematics content knowledge, pedagogical content knowledge, and knowledge of students. Consequently, the video tapes showing classroom lessons functioned as exemplars (some better than others) of communities in which making sense of mathematics was supported and developed.

For similar reasons, the opportunites to practice analysis and contemplate action with respect to teaching mathematics were better facilitated by having all students participate in shared experiences over which the instructor had some control (i.e., could choose the videos and frame the questions to be considered). The videos (rather than written cases) were used because they provided *visual* documentation of instructional events and student responses. In addition, during the course, a few selected readings (in addition to Van de Walle, 1994) were included that were chosen explicitly because they also provoked reflection and dialogue (Buschman,1993; Burns. 1996, Hankes, 1996, Watanabe, 1996), although none were actual written cases.

Teacher candidates, at the beginning of the course, wrote their mathematical autobiographies; there was evidence (See Figure 4 for examples) that, while many had experiences in higher-level mathematics and could do mathematics, many viewed mathematics learning as "teaching by telling," and several did not like and/or had mixed experiences with mathematics. Therefore, the video tapes (again, some better than others) showing classroom lessons and student interviews were intended as stimulants to personal reflection. At times, such stimulants were overt and directed in terms of the assignments given as they related to the videos; at other times, teacher candidates raised questions and concerns in class that indicated that there was, for some, a continuing element of personal reflection throughout the course.

Figure 4: Excerpts from Mathematics Autobiography

Teacher Candidate 1	Teacher Candidate 2	Teacher Candidate 3
Courses: (precollege) Algebra I, II; Geometry, Advanced Math/Trigonometry, AP Calculus; (college) Statistics Algebra was a struggle, and Algebra II was even worse. I was generally able to make "Bs", but I still had little idea of what I was doing. I was still trying to memorize and follow procedures over and over. Geometry was a breeze except for proofs. Visual figures were easy for me to work with because I could see what I was doing, and I could logically figure out processes. Proofs were harder, but I managed them fairly well because they were all about processes and the correct order of operations.	Courses: (precollege) Algebra, Geometry, Algebra II, Advanced Algebra/Trigonometry; (college) Symbolic Logic I must admit, I am not math's greatest fan. With my second major being English, I had not given a thought to mathematics in the last few years. When faced with this assignment to recount my mathematical life, my mind drew a complete blank. Mathematical life?! In my view of reality, I would not associate the words "mathematics" and "life"! Up until the eighth grade, I was a very strong student in all areas of school. However, when I hit Pre-Algebra, things started to change. All of a sudden, I started to make Cs on every one of my tests.	Courses: (precollege) Algebra, Geometry, Honors Algebra II, Pre-Calculus; (college) Symbolic Logic Math has never been my favorite subject. My experiences with math have run the gamut from fun and successful to frustrating and just plain mundane. When I changed from a public school ... to private school ... in the middle of my sixth grade year, I was placed in the lower math class based on the scores on a test I took. I feel this affected my self-esteem concerning mathematics and made me more timid about taking chances with math.
Instead of memories such as my own of breezing through the St. Jude's Math-A-Thon workbook, making low grades in calculus, and struggling to stay motivated in higher level math courses, I would rather that my own students remember that math was fund, yet a challenge; that it was not about numbers but about skills. More importantly, I hope students will remember that math is in our every day life, and that it is a very positive and necessary thing to be adept at.	I think that placing numbers and computations into different real-life situations is one way of giving math some kind of meaning to my students. How my students like math depends on how I present it to them, and I would like to learn other strategies to make them like math over the semester.	I think teaching math should accomplish the same goals from kindergarten through twelfth grade. Kids should learn basic concepts, but more importantly *how* these concepts can be applied in their world. As students progress through the grades, skills and concepts will become more complex, but should remain grounded by the ideas of meaning and application.

Research

Context

EDUC 12 Methods for Teaching Elementary School Mathematics is part of a "methods block" that teacher candidates take during the fall semester of their senior year. As part of that semester, they are in field placements one day a week in the classrooms in which they will be interning during the spring semester. There are two sections of EDUC 12, taught by different faculty; the section discussed had 20 students (18 traditional age; 2 non-traditional age; 18 female/2 male; none with a second major in mathematics). The majority of the teacher candidates were in early-grades (K-3) field placements. They began the elementary teacher education program in their junior year, taking selected child development and general teaching courses as well as a foundations/social studies methods course. They were in field-based placements at least one half day per week during their junior year. In terms of mathematics, they met general college requirements by selecting from a number of choices (See Figure 4 for examples).

As part of the class work, a number of different strategies were used to probe for responses to video "case-like" work (See Figure 3 for summary of uses). Initially, the WGBH tapes (Set 1) were used to focus teacher candidates' attention on the design and implementation of mathematics lessons, with the recognition that, at this stage, their concerns were on management and classroom structure. The first few video assignments focused primarily on lesson planning were intended to acknowledge these concerns and permit teacher candidates to talk about issues that they felt were important.

The next set of tapes (Set 2) were designed to accompany a section of the course that considered children's development of concepts related to number. These tapes were used to provide examples of the developmental issues they were reading about in Van de Walle (1994); to provide models of the CGI approach, problem types, and developmental stages that they were reading about both in Van de Walle (1994) and Hankes (1996); and to provide models of how to conduct interviews with students in preparation for their own assignment to interview students. The viewings of short segments were interwoven with other instructional activities; the major focus was on how what was being viewed tied to readings that teacher candidates had completed. In addition, teacher candidates were encouraged to try out some of the tasks with students in their field placements; when this occurred, time was taken for sharing in class as a catalyst to encourage others to experiment as well.

The third set of tapes (Set 3) accompanied a section of the course that considered the development of students' computational knowledge. The intent was to provide experiences in which teacher candidates saw the variety of children's reasoning strategies occurring in the context of whole class lessons rather than in the context of individual (often diagnostic) student interviews. An initial video involved a short segment from a longer tape considering the development of children's addition and subtraction strategies (TERC, forthcoming); only the problems being explored at the fifth grade were considered. Students were assigned the task of watching *Multiplication of two-digit numbers* (Kamii, 1990 b) in their teams of four outside class; they were given a set of questions (see Figure 5) for which they prepared written group responses. The third tape, *Developing mathematical ideas (division)* (TERC, forthcoming), was viewed and discussed over two class periods; as with the Kamii tape, teacher candidates were assigned particular students to attend to and individual student strategies were reviewed and discussed as part of the class lesson.

Figure 5 – Viewing the Kamii Tape – Multiplication

See attached description.

1. Situation 1: Second Grade, May

   - What is the problem students are solving?
   - Describe the strategy used by each child; include a summary statement about Kamii's analysis of their strategies. The children are: Jessica, Rhonda, Third child (Shaun). Fourth Child (Joanna), Courtney, and Victor.
   - Consider this problem: 5 children are each given a bag of Peanut M&M candies; each bag has 28 candies. How many candies in all?
     What makes this problem similar to the problem shown on the tape? What makes it more difficult than the problem shown on the tape?
     Describe how you think each of the children noted above might solve this problem.

2. Situation 2: Third Grade

   - What is the problem students are solving?
   - Describe the strategy used by each child; include a summary statement about Kamii's analysis of their strategies. The children are: Shannon, Ashley, and Michael.
   - Consider this problem: 5 children are each given a bag of Peanut M&M candies; each bag costs 36¢. How much do the bags cost in all?
     What makes this problem similar to the problem shown on the tape? What makes it more difficult than the problem shown on the tape?
     Describe how you think each of the children noted above might solve this problem.

3. Based on Kamii's discussion, describe how knowledge of two-digit addition develops.

4. Describe what you infer about the role of addition in learning to do multiplication from viewing this tape.

5. What questions or concerns are raised for you as a result of viewing this tape?

The fourth set of tapes (Set 4) accompanied a section of the course that considered the development of concepts related to fractions. Each of two tapes was used as a "first lesson" that provided a particular interpretation of fractions and used a particular manipulative model. *Cookies to Share* (WGBH) was an initial lesson that led to an exploration of fractions using pattern block activities (Tierney, et al., 1995). *Fractions with Geoboards* (WGBH) provided an initial lesson that led to an exploration of fractions using geoboard activities (Tierney, et al., 1995). This sequence was part of a section that looked across grade levels at both the content and possible models for the development of students' understanding of concepts related to fractions.

The last video (Set 5 - *Arrays and Fractions* (WGBH)) was viewed as part of the final examination. Teacher candidates were permitted to watch the video together and to brainstorm responses to the focus questions (See Figure 6); individuals had to write their own responses to these questions as one of the final examination questions. They were told that the course instructor believed that there were some problems with this lesson; by and large, their prior experiences had been with videos that were used to discuss what was good about teaching. This tape was deliberately included to assess how teacher candidates might evaluate a lesson that involved more obvious dilemmas of instructional practice.

Figure 6 – Final Examination Viewing Question

View the video segment showing mathematics instruction. Respond to the questions below. Provide justification and/or evidence for your responses, referencing such resources as our text, *Elementary School Mathematics*, or other materials you have read as part of this class.

a. Describe the mathematical task – the content of the lesson. Is it a worthwhile mathematical task? Explain your reasoning.
b. Describe the nature of the discourse that is taking place, considering teacher's role, students' roles, use of tools.
c. From your view from the "back of the room", comment on the overall lesson.
   - Were there any "missed or misjudged opportunities" to engage students in a "worthwhile mathematical task." or nurture student's mathematical thinking? If so, given an example and discuss.
   - What would you do for the next lesson based on what you observed in this lesson? Why?
   - Describe one way you might assess students' understanding of the content based on what you have observed. What is the purpose of this assessment?
d. What question(s) does this experience raise about teaching and learning mathematics?

Sample Teacher Candidate Responses to part (d)

Teacher Candidate #1	Teacher Candidate #2	Teacher Candidate #3
The fact I viewed this lesson the way I did, and that I realize that I am prone to teach these kinds of lessons makes me question my own ability to teach math well. It also makes me wonder when to use the constructivist approach and when to just explicitly tell students something, because I didn't like the teacher telling students something, but what if they really needed to hear it? Likewise,	One question that was raised for me from this video was when it is appropriate to use didactic teaching and when to allow the lesson to be more child-led. I feel like in this video, the teacher was a little too quick to jump in and just tell the children the answer instead of letting them piece together the answer. Therefore, I was wondering if it is wrong to lead the class didactically when talking about a	The video didn't raise any pressing questions about teaching mathematics; the thoughts I had after watching the lesson were more like things to think about or remember when teaching. First of all, I was struck by how much wait-time can affect the quality of discourse in the classroom. The teacher in the video often led the students through their answers to questions and missed

does too much exploration at one time hinder the learning process/constructivist approach? Was this the problem? And perhaps most interestingly, how much can children learn about something (fractions) while they are working with models they are not yet comfortable with?	problem or whether you should strive to let the children discover the answer to the problem by themselves.	many opportunities to build student knowledge and solidify understandings of lesson content. ... I also thought about whether the process was really the focus of this lesson, as we've learned it should be from numerous sources (Van de Walle, Marilyn Burns) and virtually every class discussion. The speed and lack of depth of this lesson reminded me more of things that were done when I was in school when the right answer was the key to learning mathematics.

A variety of data were collected. These included all course written work, journal writing tasks, and transcriptions of two sets of interviews conducted with teacher candidates during the course. Additional data are being collected during the teacher candidates' field placements during Spring, 1997. A select number of teacher candidates are being observed while teaching a lesson from the mathematics unit that each developed as part of the course and are also being interviewed about the unit and the instructional sequence that has occurred during the teaching of the unit.

Interview Study

Five teacher candidates (4 female/1 male; all traditional age) volunteered to participate in the interview study conducted during the course; they were interviewed twice, once in late October and once in early December. Teacher candidates were audiotaped and questioned about the use of the case-like videotapes, as well as their thoughts about teaching mathematics. Excerpts from both interviews, as well as from written assignments, are shown in Figures 4, 7, and 8, (and Figures 9, 10 in second document distributed). The responses and written work of three of the teacher candidates are representative of three types of reactions to these topics: the skeptical student, the astonished student, and the reflective student.

Figure 7: Excerpts of Teacher Candidate Responses to First Interview Questions

1st Interview	Teacher Candidate #1	Teacher Candidate #2	Teacher Candidate #3
What parts of the videos stand out for you?	• We would sort of wonder … were they (the students) really all doing as well as they looked to be doing on the tape • On the one hand, it was very good because you can actually see lessons being done…but on the other hand, it was to me a little artificial • With the interviews of the specific kids, that was very helpful.	• They showed how children at the same grade level can be so far apart with what they think, you know, understand. • We could see how different children construct their own ways to get solutions.	• The parts that stand out the most are probably things we discuss afterwards. • So I remember usually what the task was and then we talk a lot about the strategies that the kids use to solve the task or what they're thinking. • I think she (the instructor) does a really good job with making sure that we have set in our minds what exactly was going on from the teacher's perspective and from the perspective of what the kids were thinking.
Why do you think the instructor used the videos?	• I think she wanted to give us an idea of just what an interview is like in the first place, what kind of questions are appropriate to ask … just sort of an introduction really to the whole idea of individually assessing what a child knows. • I think it's also an introduction to how to teach some lessons that are math lessons that aren't number crunching, that aren't very blatantly number manipulation.	• Well, we got to see varied ways of teaching and I could appreciate teaching math more. • It really helped us do our own interviews … she said they (our interviews) were the best ones she's seen. • They're obviously teachers who are successful … we see how good teachers do it and how successful teachers teach their children. • It shows there are so many different ways to teach math.	• Well, it's a lot easier than all of us trekking out to a school and watching a lesson and we would be totally intruding … it would definitely not be an authentic situation. • It's also, I think, a way for her to get across to us the philosophy behind her class and what she thinks about teaching math in an actual classroom setting … it's real teachers and real kids.
Are you thinking differently about teaching math now than at the beginning of the semester?	• I don't think any differently in terms of feeling comfortable teaching math, or feeling like I want to teach math, or feeling like I'm confident enough to teach these children math. I think what's changed is my ideas about sort of how kids think about math. • (Before) I probably would have tried to funnel them into the traditional way of doing things and now I think I would say, "Well, if that makes sense to you and if you can do it consistently, go for it, use it, do whatever your want."	• Well, I wasn't very excited about, you know, having to take this math. • It's just that I always thought it was like you're up at the board and then the teacher gives you time to do this problem, and then you give them problems, and blah, blah, blah, and they're boring, and you're bored, and they're bored. • We've seen in the videos of how you have a problem and you say, "Okay, you guys solve it the way you want," and then showing different ways – that is an exciting way of teaching for me … I'm definitely going to use that because it makes sense and is more interesting to me.	• Just more letting kids construct their own strategies and their own knowledge. • We were the early '80's kind of kids … the only way we knew to divide was to do long division where you put the number here and into whatever, and then the video we watched, the kids have so many different strategies … the one kid who actually did the long division, some of the kids were like, "Well, that strategy really doesn't make sense to me." • Now I think that you can let kids make up their own ways to do things and they will get things right. They don't have to do it in the set algorithms that we've been taught. • (In August) I really didn't have any idea how I was going to teach math in my class… and now I feel like I have a lot stronger base to go from.

Do you like math?	• I never felt I liked math. I felt like I could do it, it wasn't like a stumbling block or anything for me ... but it didn't make a whole lot of sense to me.	• No, math was pretty easy, but I didn't enjoy it. But when I had good teachers, it helped.	• It's had really high ups and really, really bad downs for me, even from kindergarten through now, like, you know, college. But I feel comfortable with math. I don't feel like it's something I can't do or something I really hate.
Was there anything in the videos that will help you in other areas of teaching?	• just having kids explain and just letting kids come up until they ran out of ways to do the problem • There's definite advantages to having your peers sort of evaluate what you're doing and give you help if you need it, and feeling like you should really put your best effort there because you're going to be up in front of the class doing it.	• In the one we just watched, she shows different ways to solve the problem because she gets into different ways, like different kinds of knowledge ... so in science you can bring in all different kinds of information and it might help one child.	• Just letting the kids help each other learn and describing each other's strategies instead of the teacher always saying, "Well, Johnny did this and this and this, and does everybody understand? Okay, now we're going to move on."
Were the videos worthwhile ?	• Well, I don't think the videos are not worthwhile. • The lesson videotapes are helpful, but I always sit there and go, "I wonder what the aide is doing right now?" or, "I wonder if the teacher actually put all these things out?" • It's definitely better than sitting there and doing all the activities as if I was a student. • It's helpful, but, again, sometimes sort of frustrating that I feel like I'm forced to be watching the very specific things.	• They've put a variety in the classroom setting and it keeps us more interested ... it's definitely a good tool. • It's just different information from observing the class versus being in the (college) classroom and taking notes. • We can see these kids are understanding how, you know, why certain computations are done that way and they are understanding the concepts involved in them.	• I think the videos are definitely more effective than (lecturing or reading), especially since we discuss them ... I think the real deep discussions that we have about them ... really helps to get our thinking going in the right direction.

Figure 8: Excerpts of Teacher Candidate Responses to Second Interview Questions

2nd Interview	Teacher Candidate #1	Teacher Candidate #2	Teacher Candidate #3
How would you redesign this math methods course?	• I don't even think there was anything bad about the course or anything that I would feel needed redesigning, but I guess if I had to pick something I would say, even if we didn't watch an entire (videotaped) lesson, just a little blip about ...what we used today. • Well, as much as I really didn't like to have to sit there and have to watch them (the videos) and then I really hated having to write up a lesson plan from what I had just seen, that actually it was kind of helpful, you know, just to see how it would really work in the classroom.	• Watching the videos is good. But about making, having to do the lesson plans (from the video lessons), maybe to do just one. • Using all the manipulatives and like actually practically going through like we were the children, and how children think about things and having to solve problems, I thought, "Oh, you know, that was really good."	• I really liked exploring with the manipulatives and doing the actual problems, but maybe a little bit less of that and a little bit more discussion of ways to bring out kids' thinking, but we did a lot of that, too.
What do you think will be the challenges of teaching math?	• I guess the challenge would be for me to really be a probing, diagnostic kind of teacher. • It's going to be a pretty consistent, I think, effort for me to have to sit and, "Well, how did you get that answer? Well, how could you get it a different way? What if I changed it around and then what would you do?" • ... designing assessments that test rather than just handing out a math worksheet and seeing how many can get it right."	• Probably knowing how to begin a major topic ... you have to think about what level they're on and practically having to do a few interviews with kids ... starting from there. • And of course, that whole idea of having them create their own understanding of it, you know that's not how I was taught. You have to let go of how you think about it and open your mind to other ways and that kind of thing.	• I think in my (student teaching) classroom the kids don't talk a lot about their thinking and their strategies behind doing things ... so I think one of the hardest things will be just to move the kids from how they're thinking now about their answers to the process .. I've tried to start to integrate a little bit of , "Tell me why you did that" or "What were you thinking when you used this way to get this answer?" • I think my hardest thing is making sure that I'm addressing the whole range of, from the lowest achieving to the ones that are doing square roots in second grade like some of my kids are right now.
Would the videos be of use to teachers "in the field?"	• Well, you can always gain insight from seeing somebody else do it. • Even if I was a teacher in a school, I would learn just as much, maybe not quite as much, but a lot from a video as opposed to just stepping into someone else's classroom and sort of seeing what's going on. • I think maybe if it was made optional, I mean, I don't think I would take to it very well if it was something required because that would make me feel like it was just one extra thing to do.	• Yeah, definitely ... a lot of these videos had new ways of teaching, new strategies or whatever. • For teachers that have been teaching for a long time, just to give them another view that they've maybe never even thought of just because it's been so long since they've been in, like, an instructional setting.	• I think they could be used at workshops or anything like that, especially if they were on a certain topic ... just like examples of taking a topic a step further than teachers might think about it. • I'd definitely go to an inservice like that. I'll probably be the teacher who goes to everything. I don't have time, but I have to!

Was it helpful to have something to focus on when watching the videos?	• On the one hand it was helpful because you can see what she (the instructor) was talking about ... on the other hand, I think sometimes, not every time, sometimes it's nice to watch the video and sort of just come to your own conclusions about it. • So, depending on the objective it works well, but it's also maybe kind of a hindrance, too.	• I think it's good because ... it's easier to just focus on one thing to deal with. • You weren't so overwhelmed with trying to pick out so many details, so many different topics. • Especially when we're in groups, if maybe one person in each group is looking for a certain thing and then there's something that you missed, you know, that somebody else says that was your topic, it definitely opens it up to more like the things that you notice, but then again, there are other important things.	• If she (the instructor) says, "OK, think about the task she's asking them to do," ... I think why she's showing us the video relates a lot to what she asks us to look for. I think it helps her to get across what she's trying to teach us to do with the video.
Are you ready to teach math?	• I certainly like it a lot better and feel like I'm more qualified to teach it now than I did earlier. • Watching the videos and being in the classroom ... I watch what the teachers are doing and I say to myself, "I can do that. If I sit down and really think about what I want them to do and really think about what materials I have, what could I use, that I think I could do it given the time and energy to make that happen. I could do that." • I still feel nervous about it, but I feel much more comfortable.	• I feel like more excited about it than I was during the semester. I mean, I was clueless when I was starting out. I was so nervous because I had no idea. • I can definitely see how to ask them (students) to give the class different understandings ... so I'm excited to try that out and see how that works, because it's probably very different from how my (cooperating) teacher teaches.	• Yeah, I mean we have so many resources, and I think the basic thing was that if you don't know how to teach it, go look at any kind of books about teaching math and just see what were some ways that were used, how you think that'll work with your individual students. • I think one of the best things we've learned over all three of our courses this semester is there's so many resources out there; use them. Don't feel like you've got to be so creative and make up your own lesson every time because you're just going to be exhausted.
What questions do you still have about teaching math?	• Well, we didn't get to certain concepts, like division ... so if I find myself teaching fifth grade or fourth grade, I might be a little concerned that we hadn't done it.	• I guess I have a question about ... just how to go through all, you know I haven't really seen a whole unit. You know, how do you start the unit and how it progresses.	• There are definite questions about content, like where do you, how in depth do you go, but I think that's just a judgment on the part of the teacher and something you realize as you get into it with your class. • In class, we've touched on how to teach different topics, but we haven't really talked about what exactly would be in this if you were doing it for three weeks.

- Teacher Candidate #1 -- The Skeptical Student: This teacher candidate had a "traditional" background in mathematics, with concerns about "allowing students to have so much control over their own learning." Her initial responses to the videotapes were tentatively positive, but she remained largely skeptical about the practicality of the lessons she saw.

  "We would sort of wonder ... were they really all doing as well as they looked to be doing on the tape?" "I always sit there and go, ... 'I wonder if the teacher actually put all these things out?'"

  She was clearly impressed by viewing the interviews of individual children, seeing them as "an introduction to the whole idea of individually assessing what a child knows." By the end of the course, this skeptical teacher candidate became much more open to ways of teaching mathematics that were not consistent with her didactic mathematical background and found the videotapes helpful in bringing concepts she learned in class to life with teachers and children in the classroom. She suggested that, after introducing a concept, activity, or use of a manipulative in class,

  "even if we didn't watch an entire (videotaped) lesson, just a little blip about ... what we used today," it would reinforce the course content. "I would learn just as much ... from a video as opposed to just stepping into someone else's classroom."

  This skeptic came to believe that she could teach mathematics differently than the way she was taught.

  "I watch what teachers are doing and I say to myself, 'I can do that. If I sit down and really think about what materials I have, what could I use, that I think I could do it given the time and energy to make that happen. I could do that."

- Teacher Candidate #2 -- The Astonished Student: This teacher candidate had a limited background in mathematics and was "not math's greatest fan."

  "It's just that I always thought it was like you're up at the board and then the teacher gives you time to do this problem, and then you give them problems, and blah, blah, blah, and they're boring, and you're bored, and they're bored."

  From the first videotape she watched, this teacher candidate was amazed by the thinking and enthusiasm the students displayed.

  "It shows that by using real-life situations to teach concepts, children are much more open to learning and taking the assignment as far as they can."

  As the class progressed, this teacher candidate continued to be astonished by the achievements of the students she saw and the problem solving encouraged by the teachers.

  "We've seen in the videos of how you have a problem and you say, "Okay, you guys solve it the way you want,' and then showing different ways – that is an exciting way of teaching for me ... I'm definitely going to use that because it makes sense and is more interesting to me."

  This teacher candidate became enthusiastic about the prospect of teaching mathematics in a way she had never considered before.

  "I was clueless when I was starting out. I was so nervous because I had no idea." "(Now) I feel more excited about it!"

- Teacher Candidate #3 -- The Reflective Student: This teacher candidate had a variety of experiences with mathematics.

  "Math has never been my favorite subject. My experiences with math have run the gamut from fun and successful to frustrating and just plain mundane."

  While she acknowledged the affect of these experiences, she remained open-minded about how to teach mathematics. She viewed the videotapes as an opportunity not only to see examples of

students and teachers, but to discuss them in-depth. When asked about the value of the videotapes, she continued to stress the processing of the material.

> "I think the videos are definitely more effective than (lecturing or reading), especially since we discuss them ... I think the real deep discussions that we have about them ... really help to get our thinking going in the right direction."

Even her concerns about teaching mathematics showed a great deal of growth and reflection on what she had learned throughout the semester.

> "I think in my (student teaching) classroom the kids don't talk a lot about their thinking and their strategies behind doing things ... so I think one of the hardest things will be just to move the kids from how they're thinking now about their answers to the process ... I've tried to start to integrate a little bit of, 'Tell me why you did that' or 'What were you thinking when you used this way to get this answer?'"
>
> "I think my hardest thing is making sure that I'm addressing the whole range of, from the lowest achieving to the ones that are doing square roots in second grade like some of my kids are right now."

This teacher candidate's reflections helped her bridge the viewing of the videotapes with her own practice in the classroom.

Preliminary Findings

Based on the analysis of the data from five teacher candidates being interviewed twice and a review of selected documents of eight teacher candidates' written work, we have noted the following points:

- By the end of the course, teacher candidates expressed a desire to be accepting of a variety of student responses and to be open to more than one "right" way of solving problems.
- Teacher candidates also stressed the importance of using different strategies and probes to "get at" student thinking, both in mathematics and other subject areas they will be teaching.
- Teacher candidates cited the videotaped cases as worthwhile. Some reasons given include:
  - demonstration of teaching in real-life situations
  - demonstration of classroom management strategies
  - seeing how different children understand in different ways
  - helping the instructor "get across what she's trying to teach us to do"
  - providing a common classroom situation as a basis for group discussion
  - providing a model for conducting student interviews
  - reinforcing the belief that "you can really teach that way"
- Teacher candidates expressed concerns about what to teach daily in a long-term unit – "...what exactly would be in this if you were doing it for three weeks." Some mentioned that the videotapes gave them only a piece of the "big picture."
- Teacher candidates mentioned that they would rather not be required to write as many lesson plans based on the videotaped lessons.
- Teacher candidates appeared to evolve from didactic ideas about the teaching of mathematics toward a more student-centered philosophy. Although they often expressed nervousness and uncertainty, they uniformly indicated their desire to establish accepting, open, active, self-directed classroom atmospheres for the learning of mathematics, as well as other subjects.

Building a Research Agenda

The use of videos to provide "case-like" experiences provokes a range of questions for potential research. These include the following:

- The typical protocol for use of cases includes appropriate theoretical readings, reading of the case, writing of individual responses, group discussion, and writing of additional responses. How is such a protocol implemented using video-taped episodes of classroom practice and/or of student interviews?

- How much direction is appropriate in preparation for watching a video tape? How does this vary based on the particular video being used, the experience of the viewer, the purpose(s) for viewing, and so on?
- How are preservice and practicing teachers' responses to the use of videos as "case-like" experiences similar or different?
- How do we assess impact of such experiences on teaching practice – both for teacher candidates and practicing teachers?
- What is the nature of the interactions that occur when both written and video "case-like" experiences are used?

Clearly, there are a number of other questions that surface, all of which need attention if we are to better understand how we can assist teacher candidates and practicing teachers in their efforts to help students learn mathematics in meaningful ways.

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